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1997 J. Phys. A: Math. Gen. 30 3279

(http://iopscience.iop.org/0305-4470/30/10/008)

J. Phys. A: Math. Gen. **30** (1997) 3279–3284. Printed in the UK PII: S0305-4470(97)79872-X

Pseudo winding numbers and the spherical ansatz

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Received 26 November 1996, in final form 14 March 1997

Abstract. The path-dependent surface/time integral contribution to the topological charge in an SO(3) Yang–Mills theory is studied for paths in field space that interpolate between a background gauge field in the remote past and a gauge transform of it in the remote future. The possibility of existence of such paths along which this integral vanishes for a given initial background gauge field is related to the action of the group of gauge transformations of real, pseudo winding numbers on the physical states of the theory in the background gauge field. The analysis takes a particularly transparent form for the spherically-symmetric fields of the spherical ansatz, leading to a simple interpretation of the results.

In a previous work [1], motivated by some recent investigations of the vacuum structure of non-Abelian gauge theories [2], the behaviour of the semiclassical, physical states in a non-Abelian background gauge field defined on a non-compact space has been analysed. In this case, the group of 'large' gauge transformations [3] includes those with real winding numbers, not just those with integer winding numbers as in the case where space is compact (e.g. a 3-sphere). Particular attention has been paid to the question of whether the physical states of the theory transform non-trivially under this group of 'large' gauge transformations, no longer topologically non-trivial in this case, and whether a $\theta F\tilde{F}$ term added to the Lagrangian of the theory still reproduces the phase factors acquired by physical states undergoing such gauge transformations.

Consider an SU(2) (or SO(3)) Yang–Mills theory. It is convenient to use the 2×2 Hermitian, traceless matrices $A_\mu = eA_\mu^a\tau^a/2$, where $\tau^a(a=1,2,3)$ are Pauli matrices, so then the curvature tensor is given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}[A_\mu, A_\nu]$. The topological charge q is the integral of the second Chern character of a principal SU(2) or SO(3) bundle over spacetime

$$q = \frac{1}{16\pi^2}\int \mathrm{d}^4x\, \epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[F_{\mu\nu}F_{\rho\sigma}] = \frac{1}{4\pi^2}\int \mathrm{d}^4x\,\partial_\mu K^\mu$$

where

$$K^\mu = \epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left[\frac{1}{2}A_\nu\partial_\rho A_\sigma - \frac{\mathrm{i}}{3}A_\nu A_\rho A_\sigma\right]. \tag{1}$$

Using Stokes' theorem, we can write $q = I_1 - I_2$, where

$$I_1 = \frac{1}{4\pi^2}\int \mathrm{d}^3x\, K^0[\boldsymbol{A}]\Big|_{t=-\infty}^{t=+\infty} \qquad I_2 = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\mathrm{d}t\int_{\partial S}\mathrm{d}\boldsymbol{S}\cdot\boldsymbol{K}$$

and ∂S is the boundary of space.

The first integral I_1 is the difference between the space integrals of the Chern–Simon's form of the second Chern character evaluated at $t = \pm\infty$. It only depends on the initial and final gauge field configurations $\boldsymbol{A}(t = -\infty; \boldsymbol{r})$ and $\boldsymbol{A}(t = +\infty; \boldsymbol{r})$, respectively, but not on the actual path in field space connecting them. Moreover, if the two configurations are related by a gauge transformation

$$\boldsymbol{A}(t = +\infty; \boldsymbol{r}) = U[\boldsymbol{A}(t = -\infty; \boldsymbol{r}) - \mathrm{i}\boldsymbol{\nabla}]U^{-1} \tag{2}$$

then we have

$$I_1(U, \boldsymbol{A}) = \frac{\mathrm{i}}{24\pi^2}\epsilon^{ijk}\int \mathrm{d}^3x\ \mathrm{tr}[(-\mathrm{i}\partial_i U^{-1})U(-\mathrm{i}\partial_j U^{-1})U(-\mathrm{i}\partial_k U^{-1})U] \\ + \frac{1}{8\pi^2}\int_{\partial S} \mathrm{d}\boldsymbol{S}\cdot \mathrm{tr}[(-\mathrm{i}\boldsymbol{\nabla}U^{-1})U \times \boldsymbol{A}(t = -\infty, \boldsymbol{r})]. \tag{3}$$

The second integral I_2 is path dependent in general, hence the topological charge q does not always admit a simple interpretation unless space is a compact manifold without a boundary, in which case both I_2 and the surface integral in (3) vanish, and q reduces to the integer winding number $\nu(U)$ of the gauge transformation U. This is the usual case obtained by the one-point compactification of E^3 into S^3.

If space is not compact or has a boundary, the path-dependent integral I_2 does not necessarily vanish, and can be written in the form

$$I_2 = \frac{1}{8\pi^2}\int_{-\infty}^{\infty}\mathrm{d}t\int_{\partial S}\mathrm{d}\boldsymbol{S}\cdot\mathrm{tr}[A_0(\boldsymbol{B} - \mathrm{i}\boldsymbol{A}\times\boldsymbol{A}) - \boldsymbol{A}\times\boldsymbol{E}] \tag{4}$$

where $E_i = F_{0i}$ and $B_i = -\frac{1}{2}\epsilon_{ijk}F^{jk}$ are the non-Abelian electric and magnetic fields, respectively. Assuming that I_2 actually does vanish, and that $\mathrm{e}^{-\mathrm{i}\theta I_1}$ can be identified with the phase factor acquired by the semiclassical physical states in the background gauge field $\tilde{\boldsymbol{A}} = \boldsymbol{A}(t = -\infty, \boldsymbol{r})$ under the 'large' gauge transformations of the form

$$U_\nu(\boldsymbol{r}) = \mathrm{e}^{\frac{1}{2}\mathrm{i}\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\phi_\nu(r)} \qquad \phi_\nu(r = 0) = 0 \qquad \lim_{r\to\infty}\phi_\nu(r) = 2\pi\nu \qquad \nu \in R \tag{5}$$

thus providing a non-trivial representation of the group of such gauge transformations, I_1 must be invariant under gauge transformations of the background gauge field $\tilde{\boldsymbol{A}}$. This leads to the 'representability condition' [1]:

$$\int \mathrm{d}\Omega\ \mathrm{Tr}\{[\boldsymbol{\tau} - (\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}]\cdot\boldsymbol{\mathcal{A}}(\hat{\boldsymbol{r}})\} = 0 \qquad \int \mathrm{d}\Omega\,\mathrm{Tr}\{\tfrac{1}{2}(\boldsymbol{\tau}\times\hat{\boldsymbol{r}})\cdot\boldsymbol{\mathcal{A}}(\hat{\boldsymbol{r}})\} = 4\pi \tag{6}$$

where $\boldsymbol{\mathcal{A}}(\hat{\boldsymbol{r}}) = \lim_{r\to\infty} r\tilde{\boldsymbol{A}}(\boldsymbol{r})$, and the integrals are over the solid angle Ω. This condition is obeyed if and only if the $j = 0$ part of $\boldsymbol{\mathcal{A}}(\hat{\boldsymbol{r}})$ is gauge equivalent to a pure monopole field of twice the minimum Dirac charge value (for an SO(3) gauge group) [4]:

$$\boldsymbol{\mathcal{A}}(\hat{\boldsymbol{r}}) \simeq \tfrac{1}{2}\boldsymbol{\tau}\times\hat{\boldsymbol{r}} + [j > 0\ \text{terms}] \tag{7}$$

where j is the total angular momentum quantum number, $\hat{\boldsymbol{J}} = \hat{\boldsymbol{L}} + \hat{\boldsymbol{S}} + \hat{\boldsymbol{T}}$, and $\hat{\boldsymbol{L}}$, $\hat{\boldsymbol{S}}$, and $\hat{\boldsymbol{T}}$ are the orbital angular momentum, spin angular momentum, and SU(2) isospin operators, respectively. In this case $I_1(U_\nu, \boldsymbol{A}) = \nu$, and the $|\theta\rangle$ states provide a non-trivial representation of the group of large gauge transformations: $U_\nu|\theta\rangle = \mathrm{e}^{-\mathrm{i}\nu\theta}|\theta\rangle$.

The purpose of this work is to clarify the meaning of condition (6) and the reasons for a particular background gauge field to obey or violate it. In order to do this, the path-dependent integral I_2 will be studied in more detail. If we consider space to be E^3, and ∂S a 2-sphere of infinite radius, I_2 vanishes, for example, if $\boldsymbol{B}$ and $\boldsymbol{E}$ go to zero at spatial infinity faster than $1/r^{3/2}$, which are the conditions taken as a starting point for the topological classification of gauge transformations of time-periodic gauge fields in a non-compact space [5]. Alternatively, the integral vanishes provided that the following two conditions are satisfied.

(1) At spatial infinity, A_0 vanishes and $\boldsymbol{A}$ goes to zero at least as fast as $1/r$ up to a gauge transformation.

(2) $\boldsymbol{E}$ goes to zero at spatial infinity faster than $1/r$.

The problem now is that given an initial field configuration that obeys the first condition, there is no guarantee that there exists a path in field space interpolating between the initial field configuration at $t = -\infty$ and a gauge transform of it at $t = +\infty$ along which the electric field $\boldsymbol{E}$ has the required behaviour at spatial infinity†. There is actually no such path for any background gauge field $\boldsymbol{A}$ not obeying (6), since the gauge variation of I_2 must cancel the non-vanishing gauge variation of I_1 in order to make the whole integral gauge invariant as the integrand itself is gauge invariant. If there is a path of vanishing I_2 for some given background gauge field, we can think of the quantities

$$\frac{1}{4\pi^2}\int \mathrm{d}^3x\, K^0[\boldsymbol{A}]\bigg|_{t=-\infty} \qquad \frac{1}{4\pi^2}\int \mathrm{d}^3x\, K^0[\boldsymbol{A}]\bigg|_{t=+\infty}$$

as some kind of 'pseudo winding numbers' of the initial and final gauge field configurations, and of their difference I_1 as the 'pseudo winding number' of the gauge transformation that relates them. These winding numbers generalize the topological winding numbers of the compact case, but they have no topological significance themselves.

Consider, for example, the following path in the field configuration space

$$\boldsymbol{A}(\boldsymbol{r},t) = U(\tilde{\boldsymbol{A}} - \mathrm{i}\boldsymbol{\nabla})U^{-1} \qquad U = \mathrm{e}^{-\frac{1}{2}\mathrm{i}\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\phi(r,t)} \tag{8a}$$

where

$$\lim_{r\to 0}\phi(\boldsymbol{r},t) = 0 \qquad \lim_{r\to\infty,t\to-\infty}\phi(\boldsymbol{r},t) = 0 \qquad \lim_{r\to\infty,t\to+\infty}\phi(\boldsymbol{r},t) = 2\pi\nu. \tag{8b}$$

The evaluation of I_2 in this case is straightforward in the $A_0 = 0$ gauge making use of the explicit expression for $(-\mathrm{i}\boldsymbol{\nabla}U^{-1})U$ and $(\partial_0 U^{-1})U$:

$$(-\mathrm{i}\boldsymbol{\nabla}U^{-1})U = -\frac{1}{2}(\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})\boldsymbol{\nabla}\phi - \frac{1}{2r}\{(\boldsymbol{\tau}\times\hat{\boldsymbol{r}})(1-\cos\phi) + [\boldsymbol{\tau} - (\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}]\sin\phi\}$$

$$(\partial_0 U^{-1})U = -\frac{\mathrm{i}}{2}(\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})(\partial_0\phi).$$

For background gauge fields obeying condition (6), it turns out that I_2 vanishes if and only if

$$\frac{1}{2\pi}\int \mathrm{d}\Omega\,\mathrm{tr}\left[\left(\frac{-\mathrm{i}}{2}\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\right)\hat{\boldsymbol{r}}\cdot\boldsymbol{\mathcal{A}}\times\boldsymbol{\mathcal{A}}\right] = 1. \tag{9}$$

It can be easily seen that the above condition is also obtained if we consider the slightly more general gauge paths of the form

$$\boldsymbol{A}(\boldsymbol{r},t) = F(r,t)U(\tilde{\boldsymbol{A}} - \mathrm{i}\boldsymbol{\nabla})U^{-1} \qquad U = \mathrm{e}^{-\frac{1}{2}\mathrm{i}\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\phi(r,t)}$$

where $F(r,t)$ is a smooth function of its arguments having the limiting values

$$\lim_{r\to\infty}F(r,t) = 1 \qquad \lim_{t\to\mp\infty}F(r,t) = 1.$$

† Such a behaviour would follow from the finiteness of the energy of the field if it goes to definite limits at $t = \pm\infty$, but not for field configurations such as those given in [6] for which no such limits exist. In fact, the time-dependent, spherically-symmetric, Minkowski space solutions of the Yang–Mills equations [6] lie beyond the scope of the topological classification [4].

(An example of such a function is $F(r,t) = (r^2+t^2)/(r^2+t^2+1)$ which appears in the instanton solution [7].)

Condition (9) is satisfied, for example, by the SO(3) (unstable) monopole field of twice the Dirac charge:

$$\boldsymbol{\mathcal{A}} = \tfrac{1}{2}\boldsymbol{\tau} \times \hat{\boldsymbol{r}}. \tag{10}$$

We can in fact show that requiring the electric field $\boldsymbol{E}$ to go to zero at large distances faster than $1/r$ uniquely gives the above monopole background up to an irrelevant term. Starting with the expression for the electric field along the above path in field configuration space, we have

$$\boldsymbol{E} = -\partial_0 \boldsymbol{A} = -\partial_0[U(\tilde{\boldsymbol{A}} - \mathrm{i}\boldsymbol{\nabla})U^{-1}].$$

On using the explicit forms of $(\partial_0 U^{-1})U$ and $(-\mathrm{i}\boldsymbol{\nabla}U^{-1})U$ given above, and their commutator, we have

$$\boldsymbol{E} = U\left\{\frac{1}{2}(\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})\boldsymbol{\nabla}\partial_0\phi - \mathrm{i}(\partial_0\phi)\left[\frac{1}{2}\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}},\, \tilde{\boldsymbol{A}}\right] + \frac{1}{2r}(\partial_0\phi)[\boldsymbol{\tau} - (\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}]\right\}U^{-1}.$$

Now, since for large r, $\phi \sim 2\pi\nu + O(1/r)$, the first term in the above equation is $O(1/r^2)$. The third term is $O(1/r)$, hence, for $|\boldsymbol{E}|$ to be $o(1/r)$, this term must be cancelled by the $O(1/r)$ part of the second term (this is not possible with an initial vacuum configuration $\tilde{\boldsymbol{A}} = 0$). This gives

$$\left[\frac{\mathrm{i}}{2}\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}, \boldsymbol{\mathcal{A}}\right] = \frac{1}{2}[\boldsymbol{\tau} - (\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}] \tag{11}$$

where $\boldsymbol{\mathcal{A}}(\hat{\boldsymbol{r}}) = \lim_{r\to\infty} r\tilde{\boldsymbol{A}}(\boldsymbol{r})$. Writing (11) in component form, and doing a little bit of algebra, we find that its solution is given by (10), and that the solution is unique up to an irrelevant $\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}$ term.

It is also possible to consider another type of path in field space that interpolates between a background gauge field $\tilde{\boldsymbol{A}}$ at $t = -\infty$, and a gauge transform of it, $\tilde{\boldsymbol{A}}^U = U(\tilde{\boldsymbol{A}} - \mathrm{i}\boldsymbol{\nabla})U^{-1}$, at $t = +\infty$, namely

$$\boldsymbol{A}(\boldsymbol{r},t) = \tilde{\boldsymbol{A}} + f(r,t)(\tilde{\boldsymbol{A}}^U - \tilde{\boldsymbol{A}}) \tag{12a}$$

where $f(r,t)$ is a smooth function of position and time having the limiting values

$$\lim_{t\to-\infty} f(r,t) = 0 \qquad \lim_{t\to+\infty} f(r,t) = 1. \tag{12b}$$

In this case, we have

$$I_2 = \frac{1}{8\pi^2}\int_{\partial S} \mathrm{d}\boldsymbol{S}\cdot\mathrm{tr}[\tilde{\boldsymbol{A}} \times \tilde{\boldsymbol{A}}^U]. \tag{13}$$

It can be easily seen that the monopole background field (10) also gives a vanishing I_2 for this class of paths.

The question now is whether (9) is a new condition independent of condition (6). For the $j = 0$ part of the field, both conditions are equivalent as indicated by the fact that they are both obeyed by the same monopole background field (10), and as will be made clearer below. As for the $j > 0$ part, we should keep in mind two important facts.

(1) The $j > 0$ fields give a vanishing contribution to I_1 as follows from the defining equation (3) and the fact that $(-\mathrm{i}\boldsymbol{\nabla}U^{-1})U$ is a pure $j = 0$ function, and hence is orthogonal to the higher partial waves.

(2) Cross terms between the $j = 0$ partial waves and the higher ones give a vanishing contribution to the right-hand side of (9). This follows from the fact that the factor $(\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}$ is a $j = 0$ function and from the orthogonality of functions of different values of j.

This means that the $j > 0$ terms play no essential role in the problem: their contribution to I_1 vanishes identically, and their contribution to I_2 vanishes provided that

$$\int \mathrm{d}\Omega\, \mathrm{tr}[(\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}\cdot\boldsymbol{\mathcal{A}}^{>}\times\boldsymbol{\mathcal{A}}^{>}] = 0$$

where $\boldsymbol{\mathcal{A}}^{>}$ is the $j > 0$ part of $\boldsymbol{\mathcal{A}}$, independently of its $j = 0$ part. This justifies the detailed study of the $j = 0$ fields of the spherical ansatz [8] for which the above analysis applies particularly well. A basis for these fields is

$$u^0 = \tfrac{1}{2}(\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}) \qquad \boldsymbol{u}^1 = \tfrac{1}{2}[\boldsymbol{\tau} - (\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}] \qquad \boldsymbol{u}^2 = \tfrac{1}{2}\boldsymbol{\tau}\times\hat{\boldsymbol{r}},\ \boldsymbol{u}^3 = \tfrac{1}{2}(\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}.$$

In terms of this basis, a general, spherically-symmetric gauge field can be written as

$$A_0(\boldsymbol{r},t) = \alpha_0 u^0 \qquad \boldsymbol{A}(\boldsymbol{r},t) = \frac{1}{r}[\alpha_1\boldsymbol{u}^1 + (1+\alpha_2)\boldsymbol{u}^2] + \alpha_3\boldsymbol{u}^3 \tag{14}$$

where α_0, α_1, α_2, and α_3 are functions of r and t.

Spherical symmetry is maintained under gauge transformations of the form

$$A_\mu \to U_\phi(A_\mu + \mathrm{i}\partial_\mu)U_\phi^{-1} \qquad \text{with } U_\phi = \mathrm{e}^{\frac{1}{2}\mathrm{i}(\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}})\phi}$$

where $\phi = \phi(r,t)$. The two combinations $\alpha_1^2 + \alpha_2^2$ and $\partial_r\alpha_0 + \partial_t\alpha_3$ are gauge invariant and are related to the radial components of the magnetic and electric fields, respectively. For a vacuum (pure gauge) configuration the first quantity is equal to one and the second is equal to zero, while both quantities go to zero at large distances for a magnetic monopole configuration. In fact, condition (6), written in terms of the spherical ansatz field components, reduces to

$$\lim_{t\to-\infty} a_1 = \lim_{t\to-\infty} a_2 = 0 \tag{15}$$

where $a_\mu(t) = \lim_{r\to\infty}\alpha_\mu(r,t)$.

It is not difficult to express I_2, as given by (4), in terms of the spherical-ansatz fields, and to perform the (trivial) surface integral to obtain

$$I_2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}t[a_0(1 - a_1^2 - a_2^2) + (\partial_0 a_1)(1 + a_2) - (\partial_0 a_2)a_1]. \tag{16}$$

This reduces in the $A_0 = 0$ gauge to

$$I_2 = \begin{cases} \nu(a_1^2 + a_2^2) + \Delta a_1 & \text{for the gauge path (8)} \\ I_2 = \dfrac{\sin 2\pi\nu}{2\pi}(a_1^2 + a_2^2) + \Delta a_1 & \text{for the linear path (12).} \end{cases} \tag{17}$$

where

$$\Delta a_1 = \lim_{t\to\infty} a_1(t) - \lim_{t\to-\infty} a_1(t) = \lim_{t\to-\infty}(a_1\cos 2\pi\nu + a_2\sin 2\pi\nu - a_1).$$

The two expressions are not identical, thus reflecting the fact that I_2 is path dependent in general. However, both expressions vanish when condition (15), which is equivalent in our case to (6), is obeyed, i.e. for the monopole background (10). This of course does not mean that I_2 vanishes for *any* path starting and ending at the configuration (10) since we can still consider paths along which a_1 and a_2 do not vanish in some finite time intervals.

We can see from (15)–(17) that the allowed backgrounds are characterized by the fact that the long-range $(1/r)$ part of the gauge field $\tilde{\boldsymbol{A}}$ is invariant under the group of gauge transformations (5). This is in some sense the converse of the fact that for certain background fields (e.g. non-Abelian magnetic monopoles), the allowed gauge transformations are those under which the long-range part of the gauge field is invariant [9, 10]. It has actually been shown [9] that, for a given background gauge field $\tilde{\boldsymbol{A}}$, consistency with Gauss's law in the $A_0 = 0$ gauge requires that any generator ω of the group of gauge transformations which are not 'small' should be, up to 'small' gauge transformations, a solution of the equation $\tilde{\boldsymbol{D}} \cdot \tilde{\boldsymbol{D}}\omega = 0$, where $\tilde{\boldsymbol{D}}$ is the adjoint-representation covariant derivative relative to the background $\tilde{\boldsymbol{A}}$: $\tilde{\boldsymbol{D}} f = \boldsymbol{\nabla} f + \mathrm{i}[\tilde{\boldsymbol{A}}, f]$. Writing $\omega = \rho u^0$, and using equation (14), the equation for ω reduces to

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \rho}{\partial r}\right) = 2(\alpha_1^2 + \alpha_2^2 + \alpha_2)\rho. \tag{18}$$

It is easy to see that (18) has no solution (at all times) that goes to a finite limit as $r \to \infty$ unless a_1 and a_2 obey (15).

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